

# Mission Design for an Orbiting Volcano Observatory

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The Mission to Planet Earth initiative will require global observation of land, sea, and atmosphere and all associated phenomena over the coming years, perhaps for decades. A major phenomenon playing a major part in Earth's environment is volcanic activity. Orbital observations, including IR, uv, and visible imaging, may be made to monitor many active sites, and eventually increase our understanding of volcanoes and lead to the predictability of eruptions. This paper presents the orbital design and maneuvering capability of a low cost, volcano-observing satellite, flying in low Earth orbit. Major scientific requirements include observing as many as 10-20 active sites, either daily or every 2 or 3 days. Given specific geographic locations of these sites, it is necessary to search the trajectory space for those orbits that maximize overflight opportunities. Also, once the satellite is in orbit, it may be desirable to alter the orbit to fly over other targets of opportunity. These are active areas that are not being monitored, but which give indications of eruption, or have in fact erupted. Multiple impulse orbital maneuvering methods have been developed to minimize propellant usage for these orbital changes. Mission lifetime is assumed to be 2-3 years.

## Nomenclature

- $a$  = orbit semimajor axis
- $D$  = Earth revolutions for repeat orbit
- $h$  = circular orbit altitude
- $i$  = orbit inclination
- $J_2$  = Earth's oblateness coefficient
- $N$  = number of orbit revolutions for repeat orbit
- $P$  = orbit nodal period
- $R$  = Earth's equatorial radius
- $\mu$  = Earth's gravitational constant
- $\omega$  = nodal regression rate due to oblateness
- $\dot{\omega}$  = Earth's inertial rotation rate

## Introduction

**V**OLCANIC activity ranks with hurricanes and earthquakes with respect to energetic natural phenomena. Furthermore, unlike other phenomena, eruptions can spew out thousands of tons of gas, smoke, and ash into the air, considerably affecting the atmosphere. On average, 15 volcanoes will be in the process of erupting in a given week, with up to 60 erupting in a given year. One or two eruptions per year will occur from new volcanoes, or ones that have no previous history of activity. Over 5000 volcanoes and their eruptive characteristics, some dating back 10,000 years, have been documented.<sup>1</sup>

Understanding and monitoring volcanic activity is important for two reasons: 1) activity poses a direct threat to human life and property, and 2) vented gases and aerosols alter the atmosphere and may play an important part in climate change. Today, there is some predictive capability concerning eruptions, but data is lacking for detailed modeling. To develop a suitable data base, ground stations have to be numerous and are difficult to place in remote areas. Aircraft overflights can be dangerous and expensive to perform daily. Large-scale parametric measurements, such as imaging, heat

flow, temperature, and outgassing content can be more easily obtained from orbit, particularly with a dedicated satellite.<sup>2</sup>

## Orbiter Mission Concept

The advantages of observing volcanoes from orbit have been known for some time, and useful measurements have been made from many satellites, including the Landsat Thematic Mapper.<sup>3</sup> The possibility of flying a dedicated satellite to observe volcanoes drew encouragement when in 1988 NASA's Space and Earth Science Advisory Committee (SESAC) formally identified the need for a "strategy for incorporating small, flexible missions, particularly those with rapid response times, in the overall program for the earth sciences."<sup>4</sup>

Soon after this report was issued, David Pieri of the Jet Propulsion Laboratory (JPL) proposed a candidate mission for this program called the Orbiting Volcano Observatory (OVO). It would carry a single instrument, i.e., an 8-in. telescope that would have the capability for uv and mid-IR spectrometry as well as imaging. The telescope itself would require a steering capability, and uplinked ground commands would permit real-time tracking of a target area during overflight.

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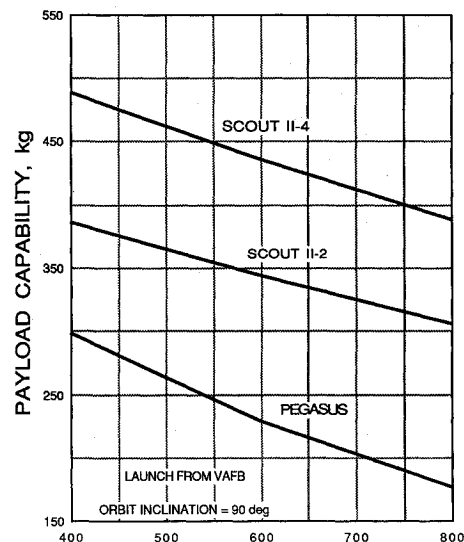


Fig. 1 Launch vehicle capability.

**Table 1** Volcanic sites and results for 1- and 2-day 70-deg repeat orbits

Name	Latitude/ longitude	Location	View parameters <sup>a</sup>	
			1 day	2 days
Mauna Loa	19.5/204.4	Hawaii	3.2A	2.7A
Kilauea	19.4/204.7	Hawaii	6.9A	5.5A
Etna	37.7/15.0	Italy	12.D	13AD
Stromboli	38.8/15.2	Italy	5.6D	7.7AD
Vesuvius	40.8/14.4	Italy	2.3D	2.3AD
Augustine	59.4/206.6	Alaska	4.8AD	0.2AD
Kliuchevskoi	56.2/160.8	Kamchatka	28A	10AD
Sakura-jima	31.6/130.7	Kyushu	—	7.2AD
Lascar	-23.4/292.3	Chile	—	17A
Santiaquito	14.8/268.5	Guatemala	28D	30A
Agung	-8.3/115.5	Indonesia	—	—
Rabaul	-4.3/152.2	New Guinea	9.8AD	29D
Erebus	-77.6/167.3	Antarctica	—	—
Poas	10.2/275.8	Costa Rica	28A	0.1AD
Arenal	10.5/275.3	Costa Rica	23A	3.7AD
Krafla	65.7/343.3	Iceland	1.3AD	3.0AD
Niragongo	-1.5/29.2	Central Africa	5.1AD	15D
Ertale	13.6/40.7	Ethiopia	—	1.0AD

<sup>a</sup>These columns give spacecraft view angle in degrees, ascending orbit view (A) and descending orbit view (D). Spacecraft view angle is restricted to 30 deg.

For these small missions, the science payload can be about 20–30% of the total spacecraft mass delivered to orbit. Typical performance capabilities of small launch vehicles are shown in Fig. 1 for two advanced Scout versions and Pegasus. A reasonable minimum spacecraft mass for this mission is about 250 kg. For Pegasus, this limits the altitude to about 550 km. Both Scout versions can do much better than this, delivering over 300 and 400 kg, respectively, for a 750-km orbit. Much higher altitudes would reduce payload capability and also cause concern regarding Van Allen belt radiation effects. Note that Fig. 1 applies to polar orbits, so that there will be a small payload increase for a lower 70-deg inclination orbit. Science payload mass for these launch vehicles can range from 50 to 120 kg.

### Science Objectives and Instrument Selection

The instrument requirements for this mission will naturally depend on the desired science objectives. The primary science objectives are the following:

- 1) Monitor the emission of gases and particulate materials into the atmosphere, to the degree necessary to determine quantity, distribution, and composition.
- 2) Monitor the behavior of selected volcanoes to enhance knowledge of precursor phenomena, magma delivery, and the mechanics of eruptions, in order to enhance our understanding of energy and material release for a variety of tectonic phenomena.

Based on these objectives, an 8-in. pointable telescope similar to the one currently operational under the Department of Defense (DOD) Delta Star program, and developed by A. Lane of JPL, has been proposed as the instrument concept. This proposal to fly a single but capable instrument is consistent with the small missions philosophy recommended by SESAC.

For use in a volcano observing mission, the derived instrument would need to include uv, IR, and visible bandpasses with approximately the following characteristics: 1) uv focal plane: 20–40 m/pixel; uv imager: 2 bands at 0.28 and 0.31  $\mu\text{m}$ ; and uv spectrometer: 2  $\text{\AA}$ /pixel resolution, 0.21–0.36  $\mu\text{m}$  (for  $\text{SO}_2$  and other gas spectrometry). 2) Visible focal plane: 40–60 m/pixel; and visible imager: one broadband in visible range, 0.4–0.7  $\mu\text{m}$ . 3) Near IR (NIR) focal plane: 60–80 m/pixel; and NIR imager: 5–10 bands between 0.7 and 2.5  $\mu\text{m}$  and 5–10 bands between 2.5 and 5  $\mu\text{m}$  (thermal radiometry).

To include the 2.5–5  $\mu\text{m}$  bandpass for the NIR will depend critically on detector efficiency and cooler technology, in

terms of designing to the Scout/Pegasus performance envelope. Also, it would be highly desirable to test this instrument concept with an aircraft flight simulation.

### Orbit Selection

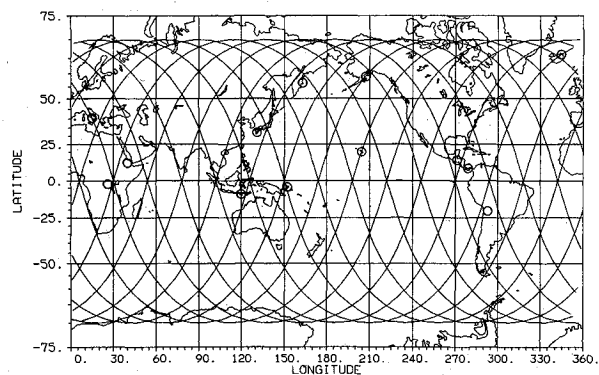
Orbit selection, for the purpose of observing a maximum number of active sites in a 1- or 2-day period, can be made based on three possible approaches:

- 1) Choose a 1- or 2-day repeat orbit (see Appendix for a discussion of repeat orbits), and find the altitude, inclination, and node location that maximizes the number of potential sites which can be viewed.
- 2) Select a 1- or 2-day Sun-synchronous repeat orbit (which will be near polar and retrograde), and find the node location that maximizes the number of sites viewed.
- 3) Select a Sun-synchronous repeat orbit, with a longer 16-day repeat period and higher altitude, so that all potential sites may be viewed within a 1-, 2-, or 3-day period, regardless of where they are located.

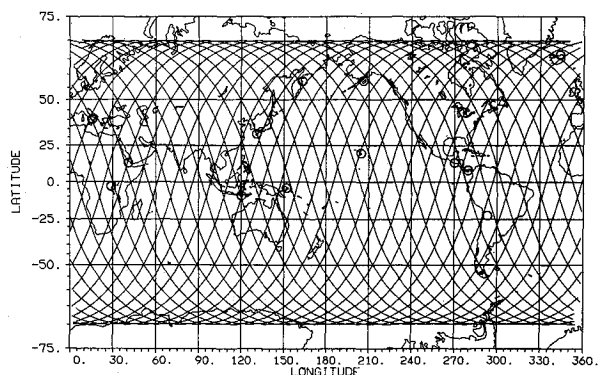
The advantages of the first and second options are that greater importance is placed on fewer sites, and that these sites are viewed nearly directly underneath the orbit. Also, higher resolution is obtained at the lower altitude (for the 1-day repeat orbits). The first option frees the inclination for maximizing the number of sites viewed, but is subject to a day/night viewing cycle. The second option is Sun-synchronous and therefore views the sites at the same local time of day (fixed Sun location) each day. The advantage of the third option is that all sites are viewed to some extent, although the viewing angle may be larger and the viewing distances longer. Also, since the orbit is Sun-synchronous, each site will be viewed at the same time of day, which may be desirable for uniform data gathering.

#### Repeat Orbits: Selection 1

Considering the first approach, a list of the most active sites was requested from the volcanologists and is given in Table 1.



**Fig. 2** Groundtrack for the 1-day repeat period 70-deg inclination orbit.



**Fig. 3** Groundtrack for the 2-day repeat period 70-deg inclination orbit.

The first part of this table presents the geographic latitude and longitude of each site, in addition to its continent or country. Also listed in Table 1 are the results of a preliminary search for the orbits maximizing the number of sites viewed for 1- and 2-day repeat orbits. The parameters involved in this search are the inclination and location of the ascending node. The altitude is fixed by the requirement for a repeat orbit.

A computer graphics program developed at JPL is well suited for this study. It runs on a Macintosh computer, has orbital elements as inputs, and produces a graphical representation of Earth with the selected sites indicated and the groundtracks overdrawn. A user can change any orbital element, run the program, and see the effects on the groundtrack within a couple of minutes. A detailed printout of trajectory related data is also available. Figures 2 and 3 show the groundtracks of the orbits that were finally chosen.

At low altitudes, for the first two approaches mentioned above, only a single 1-day repeat orbit is available. This is one that makes exactly 15 orbits in one Earth revolution (relative to the ascending node, see Appendix). For an inclination of 70 deg, this altitude is approximately 518 km. The next one higher would make 14 orbits in one Earth revolution, but this altitude is above 800 km. Similarly, for a 2-day repeat orbit, the optimum altitude occurs for 29 orbital passes in two Earth revolutions. For this case, the altitude is 678 km. Other 2-day repeat orbits are either above 800 km or below 400 km. The advantage of the 2-day repeat orbit is that there are nearly twice as many groundtracks over the Earth as for the 1-day orbit, which increases the probability of viewing a particular site.

The chosen inclination of 70 deg for the first approach represents nearly the lowest that allows one to see the sites listed in Table 1. It excludes only Erebus in Antarctica, whose latitude is about 78 deg (the orbit inclination must naturally be greater than the latitude). A higher near-polar inclination may be expected to result in a decrease in the number of viewed sites because about 20% of the orbit time would be spent above 70 deg north and south latitudes where only one site, Erebus, is located. As for the orbit node longitude, the groundtrack (ascending node input) has been shifted to the left or right to fly over as many sites as possible. The off-vertical spacecraft view angle given in Table 1 is directly related to the lateral (or cross-track) distances shown in Figs. 2 and 3.

In spite of the open spaces between the groundtracks shown in Fig. 2, the 1-day repeat orbit does a fair job of observing the given volcanic areas. One element remains, however, which

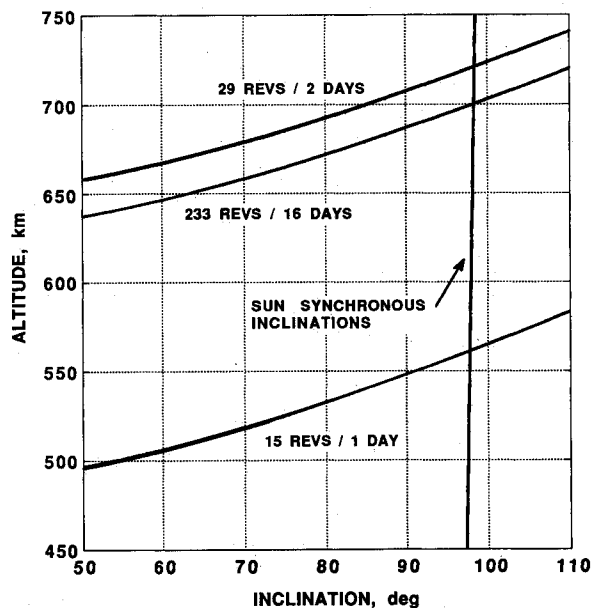


Fig. 4 Altitude vs inclination for repeat orbits.

Table 2 Repeat orbit solutions for 70-deg and Sun-synchronous inclinations

	70-deg inclination		Sun-synchronous		
	15	29	15	29	233
Orbits	15	29	15	29	233
Repeat, day	1	2	1	2	16
Altitude, km	518	678	561	720	705
Inclination, deg	70	70	97.6	98.3	98.0

degrades this observation pattern, and this is the day/night cycle. Nodal regression of the orbit, as well as Earth's motion about the sun, causes the orientation of the orbit relative to the Sun to change daily. In fact, the orbit relative to the Sun makes one complete revolution in about 100 days, or more than three times a year. Because of this orbital motion, there will be periods of time every 50 days, and lasting a few days, when the spacecraft is near sunset or sunrise. These will be times when imaging, in particular, will be degraded. Counteracting this day/night effect is the fact that some sites are overflown on both the ascending and descending portion of the orbit. Thus, if the ascending portion is in the dark, then the descending portion will be in daylight.

Comparing the effectiveness of the 1- and 2-day repeat orbits shown in Table 1, the latter obviously has the advantage. Ten of the sites are viewed on the ascending and descending portions of the orbit, whereas only four are viewed for the 1-day repeat orbit. For those sites not viewed on both sides of the orbit, the viewing will be in darkness about half the time.

#### Sun-Synchronous Orbits: Selection 2

The orbit design presented for the first approach places emphasis on maximizing the number of volcanic sites viewed directly below the orbiter on a 1- or 2-day period. In the selection, little consideration is given to the day/night cycle problem. The fact is, however, that daylight viewing is preferred, even though night viewing can offer some advantages at longer wavelengths. An alternative approach, then, would impose continuous daylight viewing as a major constraint on the orbit selection, which requires that the OVO be in a Sun-synchronous orbit; that is, be in an orbit whose node advances at the same rate as the mean Sun around the Earth, which is about a degree per day. This nodal advance is possible because of the effect of Earth's oblateness on the orbit, and the nodal rate for circular orbits is given by Eq. (A2) of the Appendix. This equation provides a relation between inclination and altitude for a required nodal rate, which in this case is 0.9856 deg/day.

On the other hand, the condition for a repeat orbit provides a relationship between altitude and inclination. That is, given some inclination, and the fact that groundtracks must fall over exactly the same paths after  $N$  orbits and  $D$  Earth revolutions, there will be a discrete altitude requirement for a specific inclination. Figure 4 presents this relationship as well as the analogous relationship for Sun-synchronous orbits, and the intersections of these curves are the solutions in the altitude range of interest. Numerical data is given in Table 2. Included in this table are also the repeat orbit solutions for the 70-deg inclination cases.

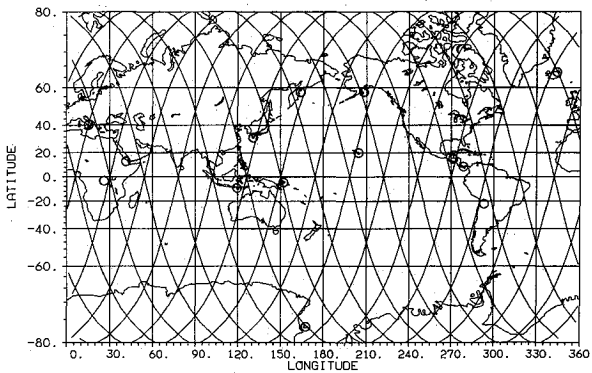
The second option, then, repeats that of the first but for Sun-synchronous orbits. Table 3, similar to Table 1, presents these results. Here, it is seen that the descending orbit provides the best coverage, therefore launch should be such that this portion of the orbit is on the sunlit side.

Being Sun-synchronous, the orbit will then remain in this Sun relative orientation during the life of the mission. The groundtrack for the 1-day repeat period is shown in Fig. 5. Note that the viewing for the 1-day repeat period is comparable to the 70-deg inclination case shown in Fig. 2. However, as indicated in Table 3, the 2-day Sun-synchronous orbit is a considerable improvement over the 1-day orbit, since all but two sites are visible on the descending orbit pass.

**Table 3 Volcanic sites and results for 1- and 2-day repeat orbits having Sun-synchronous inclinations**

Name	Latitude/ longitude	Location	View parameters <sup>a</sup>	
			1 day	2 days
Mauna Loa	19.5/204.4	Hawaii	2.8D	2.4D
Kilauea	19.4/204.7	Hawaii	6.1D	5.1D
Etna	37.7/15.0	Italy	13D	27AD
Stromboli	38.8/15.2	Italy	14D	27AD
Vesuvius	40.8/14.4	Italy	23D	19AD
Augustine	59.4/206.6	Alaska	2.2A	3.5D
Kliuchevskoi	56.2/160.8	Kamchatka	0.1A	17A
Sakura-jima	31.6/130.7	Kyushu	—	14D
Lascar	- 23.4/292.3	Chile	11D	12D
Santiaquito	14.8/268.5	Guatemala	6.7A	26AD
Agung	- 8.3/115.5	Indonesia	—	—
Rabaul	- 4.3/152.2	New Guinea	8.3AD	24D
Erebus	- 77.6/167.3	Antarctica	6.5AD	2.0AD
Poas	10.2/275.8	Costa Rica	17D	5.8D
Arenal	10.5/275.3	Costa Rica	11D	10D
Krafla	65.7/343.3	Iceland	6.1AD	2.0AD
Niragongo	- 1.5/29.2	Central Africa	26AD	26AD
Erta ale	13.6/40.7	Ethiopia	—	5.8AD

<sup>a</sup>These columns give spacecraft view angle in degrees, ascending orbit view (A), and descending orbit view (D). Spacecraft view angle is restricted to 30 deg.



**Fig. 5 Groundtrack for the 1-day repeat period Sun-synchronous orbit.**

**Sun-Synchronous Orbits: Selection 3**

Consider now the third approach, or the longer 16-day Sun-synchronous repeat orbit. This repeat orbit will be independent of specific site locations. This period is long enough so that, at some time during the 16 days, the spacecraft will view every spot on Earth. Since there will be 233 orbit revolutions in 16 Earth revolutions, the adjacent crossings on the equator will be 360/233 or 1.545 deg apart. From an altitude of 705 km, the spacecraft view angle to the adjacent ground-track will be 13.67 deg. This means that the spacecraft can view any site within 7 deg at some time during a 16-day period.

For daily viewing, it is necessary to look at consecutive orbit crossings, where the spacing on the equator is  $2\pi P/24$  or 24.72 deg, where  $P$  is the nodal period. For the worst case, where the site is exactly between these passes, the spacecraft view angle would have to be almost 60 deg. At this angle, the view of the Earth is very slanted, in fact about 70 deg from vertical at the site.

During a typical 16-day period, the daily spacecraft view angle for a site that passes directly under it on day zero has been computed and is shown in Fig. 6. Since this angle is different for sites at different latitudes, three cases have been chosen: 0, 40, and 60 deg. As expected, the higher latitude sites can be viewed more often, or at least with a lower spacecraft viewing angle. As noted above, if this angle is nearly 60 deg, then all sites can be viewed each day. If this angle is restricted to 30 deg, say, then the viewing pattern will be as shown in Table 4.

Comparing this result with the specific volcanic sites listed in Tables 1 and 3, only four sites have latitudes near 60 deg or more. Also, the three in Italy have latitudes near 40 deg. The remaining 11 have lower latitudes, and hence poorer viewing for this orbit option.

**Maneuver Analysis**

Once in orbit, the OVO spacecraft will require propellant for three functions: 1) orbital corrections including atmospheric drag makeup, 2) attitude control, and 3) orbit shifts to view targets of opportunity. Not a great deal need be said about uses 1 and 2. The chosen altitudes are all high enough that drag makeup propellant requirements are not a significant burden on the spacecraft design and operation. For example, at the lowest altitude of 518 km for the 70-deg 1-day repeat orbit, the drag can reduce the orbit altitude by about 20 m/day, or 140 m/week. In this time, the period of the spacecraft would decrease by 0.174 s, causing the total time for 105 orbits to be reduced by approximately 9 s. With the Earth rotating under the orbit at a rate of  $0.734 \times 10^{-4}$  rad/s, the nodal crossing will then be located 4.2 km further east in longitude. The spacecraft view angle to a specific equatorial site, then, will be up to 0.5 deg greater (or less) than the previous week. These variations should pose no problems in performing mission goals.

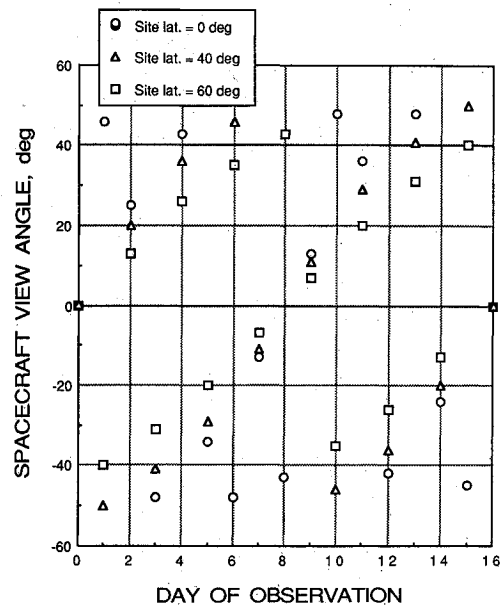
The propulsion requirements for drag makeup are also small. In terms of velocity changes, two are needed: one to raise apogee to a desired altitude, and the other to circularize the orbit once the spacecraft arrives there. The sum of these two burns can be approximated by the relation

$$\Delta V = \pi \Delta h / P \tag{1}$$

where  $\Delta h$  is the desired altitude change in the circular orbit, and  $P$  is the initial orbit period. In this case the period is 5702 s and  $\Delta h$  is 140 m. Then,  $\Delta V$  is 0.077 m/s.

**Table 4 Sixteen-day repeat viewing schedule for sites at different latitudes**

Latitude, deg	Days viewed for S/C view angle of 30 deg
0	0,2,7,9,14,16
40	0,2,5,7,9,11,14,16
60	0,2,3,4,5,7,9,11,14,16



**Fig. 6 Viewing opportunities for the 16-day Sun-synchronous repeat orbit.**

It is important that altitude corrections be performed often, such as once per week, because of the quadratically accumulative effects with respect to time. That is, not only does the orbit period deviate linearly with time, but also the drift increases with the number of orbits (i.e., time). For example, if a correction were made only once per month, the above calculation would yield a nodal crossing longitude drift of 78 km, or spacecraft viewing angle change of 8.5 deg. The altitude makeup requirement varies linearly, however, and would only be 0.33 m/s.

For the attitude control function, the propellant requirements for a 3-year mission have not been determined in this analysis, but are expected to be a small fraction of the total capability.

A useful capability for the OVO spacecraft (function 3) would be the ability to shift its orbit to view a site with new or increased volcanic activity. If this new site fortuitously lies on or near the OVO nominal groundtrack, then no orbit shift would be required. If the site does not, then a decision must be made to determine if the spacecraft orbit should be shifted to observe it.

Looking at the candidate orbit designs, the option using the Sun-synchronous 16-day repeat orbit would need no changes. By definition, it will see any given point on Earth with the same frequency as one of the nominal sites having the same latitude. The data shown in Fig. 6 and Table 4 still apply. This orbit option, however, may be least attractive for targets of opportunity since frequent (daily, if possible) viewing would be very desirable. On the other hand, it is the most attractive option if it is also desirable not to disrupt the viewing of the mission selected sites.

In the case of the 1- or 2-day repeat orbits, the principle question is: What is the maneuver strategy to be used, and subsequently the propellant cost, to shift the orbit node east or west by a specific number of degrees and still maintain the inherent repeat orbit characteristics? For example, it is possible to perform a single out-of-plane maneuver at a high latitude to shift the node location without altering other orbit characteristics. This maneuver will be located at the intersection of the existing groundtrack and the desired groundtrack, and is approximated by

$$\Delta V = 2V_c \sin(\Delta L / 2) \quad (2)$$

where  $\Delta L$  is the desired longitude shift, and the circular velocity  $V_c$  is approximately 7.6 km/s. For a 10-deg change, then,  $\Delta V = 1.3$  km/s, which is much too large for this mission (and probably any other).

An alternate and more efficient strategy is needed, and the only one acceptable depends on changing the nodal period of the orbit to allow the node (and hence the groundtrack) to drift to the desired point (east or west) over a period of time. Then, when the desired node location is reached, the period is returned to its original value. The longitude shift will be given by

$$\Delta L = N(\omega - \Omega)\Delta P \quad (3)$$

and the duration by

$$\Delta t = ND(P + \Delta P) \quad (4)$$

The drift rate is then

$$\Delta L / \Delta t = (\omega - \Omega)\Delta P / P \quad (5)$$

The longitude shift required may be as large as half of the distance between adjacent nodes, which is 24 deg for the 1-day repeat orbit, and 12.41 deg for the 2-day repeat orbit.

Assuming the 1-day repeat orbit (70-deg inclination), the maximum shift required would be 12 deg, or half of the adjacent node distances. However, since the spacecraft view

can be 30 deg, the whole shift need not be made (i.e., a 30-deg view angle can see 2.5 deg laterally from the groundtrack). Assuming a 10-deg shift, then, Eqs. (3) and (4) give  $\Delta P = 148.6$  s,  $\Delta t = 87700$  s (westward motion), and  $\Delta t = 83300$  s (eastward motion), where the lower value reflects the fact that the period has been decreased. The period is related to orbital radius by

$$P = 2\pi \sqrt{\frac{a^3}{\mu}} \quad (6)$$

then,

$$\Delta P = 3\pi \sqrt{\frac{a}{\mu}} \Delta a \quad (7)$$

Equation (7) shows that the altitude change ( $\Delta a$ ) would have to be 120 km. Then, by Eq. (1), the velocity requirement  $\Delta V = 66$  m/s, which would have to be doubled to return the orbit to its original altitude. Then, after viewing the new site, say for several weeks, the orbit would have to be shifted to its nominal groundtrack which would double the  $\Delta V$  requirement again, yielding a total of 264 m/s.

A reasonable maximum allocation for each target of opportunity is about 50 m/s, which would require a propellant usage of about 5 kg. This lower value could be met, for example, if the altitude change were 24 km instead of 120 km, and if the allotted drift time were increased to 5 days. The  $\Delta V$  requirement would then be about right. It should be remembered that this is an extreme case. Chances are good that the shift requirement would be closer to half of the 10 deg chosen here. Also, this is for the 1-day repeat orbit. For the 2-day repeat orbit, since adjacent nodes are half of the distance apart, the propellant requirements are also about half. The important difference here, of course, is that the new site will be seen only every other day.

## Conclusions

This analysis has investigated three options for the orbit design of a dedicated volcano observing satellite. The first, or 70-deg inclination option, provides good coverage, but is subject to an undesirable 100-day repeating day/night cycle. This problem is alleviated by going to the higher altitude, Sun-synchronous orbit option, having a 1- or 2-day repeat period, without greatly compromising the viewing of selected sites. The third option maintains the Sun-synchronous orbit, but moves to a 16-day repeat orbit, which can view any Earth location every third day (on average), to nearly every day for those sites at higher latitudes. This option has the advantage that targets of opportunity are automatically included in the observation schedule.

It is concluded that the Sun-synchronous 1- or 2-day repeat orbit would be the best choice. This option would provide frequent and favorable viewing conditions for the sites selected. Also, targets of opportunity could be accommodated with reasonable wait times in the groundtrack shift, for reasonable amounts of propellant expenditure. More detailed analysis of this Earth explorer application must await better definition of science goals, and instrument and spacecraft designs.

## Appendix: Calculation of Repeat Orbits

Repeat orbits are orbits whose groundtracks fall exactly over the same paths after  $N$  orbits and  $D$  Earth revolutions. If these numbers are relative primes (no common divisors, as is usually assumed), then the ascending tracks after  $D$  revolutions will lie uniformly around the equator. That is, after  $N$  orbits, they will be  $2\pi/N$  rad apart. Also, since the total number of radians traversed is  $2\pi D$ , then the angle between consecutive equatorial crossings is  $2\pi D/N$ .

For a given circular orbit (near zero eccentricity), the time between equatorial crossings, taking into account first order nodal regression due to oblateness, is

$$P = 2\pi\sqrt{\frac{a^3}{\mu}} \left[ 1 - \frac{3}{2}J_2\left(\frac{R}{a}\right)^2 (4 \cos^2 i - 1) \right] \quad (\text{A1})$$

and is called the "nodal period."

The actual position of the node on the equator will depend on the inertial rotation rate  $\omega$  of the Earth and nodal regression due to oblateness. The nodal regression rate is given by

$$\Omega = -\frac{3}{2}J_2\sqrt{\frac{\mu}{a^3}}\left(\frac{R}{a}\right)^2 \cos i \quad (\text{A2})$$

In the time of one nodal period, the Earth will move forward by  $P\omega$  radians, and the orbit node by  $P\Omega$ . Then

$$P(\omega - \Omega) = \frac{2\pi D}{N} \quad (\text{A3})$$

Substituting Eqs. (A1) and (A2) into Eq. (A3) gives the following polynomial in the single unknown  $a$ :

$$A_1a^{5.5} + A_2a^4 + A_3a^{3.5} + A_4a^2 + A_5 = 0$$

This equation is derived (with coefficients) and presented in Ref. 5, where it is suggested that the value for  $a$  be solved for by a numerical technique.

An alternate approach, presented here, makes use of analytic derivatives, and provides a unique and rapidly converging solution, particularly where the altitude range is narrow. Between 400 and 800 km, the variation for  $P$  and  $\Omega$  can be assumed to be linear. With that assumption, Eq. (A3) becomes

$$PQ = K$$

where

$$Q = \omega - \Omega \quad \text{and} \quad K = 2\pi D/N$$

and

$$(P_o + \Delta P)(Q_o + \Delta Q) = K$$

$P_o$  and  $Q_o$  are values of  $P$  and  $Q$  computed at an arbitrary altitude, say 500 km. Expanding gives

$$P_oQ_o + P_o\Delta Q + Q_o\Delta P + \Delta P\Delta Q = K$$

where, neglecting the higher-order term  $\Delta P\Delta Q$ , one has

$$\Delta P = \frac{dP}{da}\bigg|_o \Delta a \quad \text{and} \quad \Delta Q = \frac{dQ}{da}\bigg|_o \Delta a$$

Indicating derivatives by primes yields

$$P_oQ_o + P_oQ'_o \Delta a + Q_oP'_o \Delta a = K$$

or

$$\Delta a = \frac{K - P_oQ_o}{P_oQ'_o + Q_oP'_o} \quad (\text{A4})$$

Now, rewriting  $P$  and  $Q$  [Eqs. (A1) and (A2)] gives

$$P = C_1a^{3/2}(1 - C_2a^{-2}) \quad (\text{A5a})$$

$$Q = \omega + C_3a^{-3.5} \quad (\text{A5b})$$

Then

$$P' = \frac{3}{2}C_1a^{0.5} + \frac{1}{2}C_1C_2a^{-1.5} \quad (\text{A5c})$$

$$Q' = -3.5C_3a^{-4.5} \quad (\text{A5d})$$

where

$$C_1 = \frac{2\pi}{\sqrt{\mu}} \quad (\text{A6a})$$

$$C_2 = \frac{3}{2}J_2R^2(4 \cos^2 i - 1) \quad (\text{A6b})$$

$$C_3 = \frac{3}{2}J_2\sqrt{\mu}R^2 \cos i \quad (\text{A6c})$$

For the OVO mission, for example, an average altitude selection can be 500 km. Then, choosing an inclination of 70 deg, and the following constants,  $\mu = 398,600 \text{ km}^3/\text{s}^2$ ,  $J_2 = 0.00108263$ ,  $R = 6378.14 \text{ km}$ , and  $\omega = 7.292115 \times 10^{-5} \text{ rad/s}$ .

We find from Eqs. (A5) and (A6) that  $C_1 = 9.95202 \times 10^{-3}$ ,  $C_2 = -35151.48$ ,  $C_3 = 14265261.71$ ,  $a_o = 6878.14 \text{ km}$ ,  $P_o = 5681.203$ ,  $Q_o = 7.344976 \times 10^{-5}$ ,  $P'_o = 1.237743$ , and  $Q'_o = -2.689851 \times 10^{-10}$ .

Substituting these into Eq. (A4),

$$\Delta a = \frac{K - 0.41728297}{8.938376 \times 10^{-5}}$$

or

$$\Delta a = \Delta h = 11,187.71K - 4668.44 \quad (\text{A7})$$

This equation may now be used to approximate the  $\Delta h$  from 500 km for any value of  $K$ .

For a repeat orbit, where  $N = 15$ ,  $D = 1$ , then  $K = 2\pi D/N = 0.418879$ ,  $\Delta h = 17.85 \text{ km}$ , or  $h = 517.85 \text{ km}$ . This altitude may be used as a good approximation, or to recompute the values of  $P$ ,  $Q$ , and their derivatives to get a new value of  $\Delta a$ . The converged value is 517.85 km, or the same as the first value.

If Eq. (A7) is used for  $N = 29$  and  $D = 2$ , then  $K = 0.433323$ ,  $\Delta h = 179.45 \text{ km}$ , or  $h = 679.45 \text{ km}$ . The converged value is 678.23 km.

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